Why the Cosmological Constant Problem is Hard

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Abstract

We consider a recent proposal to solve the cosmological constant problem within the context of brane world scenarios with infinite volume extra dimensions. In such theories bulk can be supersymmetric even if brane supersymmetry is completely broken. The bulk cosmological constant can therefore naturally be zero. Since the volume of the extra dimensions is infinite, it might appear that at large distances one would measure the bulk cosmological constant which vanishes. We point out a caveat in this argument. In particular, we use a concrete model, which is a generalization of the Dvali-Gabadadze-Porrati model, to argue that in the presence of non-zero brane cosmological constant at large distances such a theory might become effectively four dimensional. This is due to a mass gap in the spectrum of bulk graviton modes. In fact, the corresponding distance scale is set precisely by the brane cosmological constant. This phenomenon appears to be responsible for the fact that bulk supersymmetry does not actually protect the brane cosmological constant.

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I. THE MODEL

Recently it was pointed out in [1,2] that, in theories where extra dimensions transverse to a brane have infinite volume [3–7], the cosmological constant on the brane might be under control even if brane supersymmetry is completely broken. The key point here is that even if supersymmetry breaking on the brane does take place, it will not be transmitted to the bulk as the volume of the extra dimensions is infinite [1,2]. Thus, at least in principle, we should be able to control some of the properties of the bulk with the unbroken bulk supersymmetry. In particular, vanishing of the bulk cosmological constant need not be unnatural[1].

Then the "zeroth-order" argument goes as follows [1,2]. Let us for definiteness focus on the case of the codimension one brane embedded in D-dimensional space-time. At least naively, at large (enough) distances, which are precisely relevant for the discussion of the cosmological constant, the theory is expected to become D-dimensional. In particular, the laws of gravity, such as Newton's law, are expected to become D-dimensional at such distances. If so, a brane world observer would then really be measuring the D-dimensional (and not $(D-1)$-dimensional) cosmological constant, which vanishes by bulk supersymmetry. One therefore might expect that the cosmological constant on the brane might somehow also vanish regardless of brane supersymmetry.

The above argument might *a priori* have (at least) two possible caveats. First, it is not completely clear what is the relation between the $(D-1)$-dimensional cosmological constant and the D-dimensional one. More precisely, one would like to see a bit more explicitly how bulk supersymmetry controls the cosmological constant on the brane. Note that the latter is certainly well-defined as being proportional to the curvature on the brane. In the following we will argue that bulk supersymmetry in a concrete model of the aforementioned type does not actually imply vanishing of the brane cosmological constant. Second, even though the extra dimension has infinite volume, *a priori* it is not completely obvious why the theory should remain D-dimensional above some large crossover distance scale r_0. Thus, one can imagine a scenario where the theory is effectively $(D-1)$-dimensional at length scales $r \ll r_0$, it becomes D-dimensional at intermediate scales $r_0 \ll r \ll r_*$, and it then again becomes $(D-1)$-dimensional at larger scales $r_* \ll r$. If so, the natural bound (in the General Relativity conventions) for the cosmological constant $\widetilde{\Lambda}$ on the brane would be $|\widetilde{\Lambda}| \lesssim 1/r_*^2$. As we will argue in the following, this is precisely what appears to be the case in the model we discuss in this paper[2]. In fact, as we will see, in, say, the Dvali-Gabadadze-Porrati model [7] there is a mass gap in the spectrum of bulk graviton modes if the cosmological constant on the brane is positive. This then explains how come the aforementioned "zeroth-order" argument does not really apply as the theory is effectively

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[1]Note that *a priori* we could have negative cosmological constant consistent with bulk supersymmetry. However, in the presence of supersymmetry various ways are known for ensuring vanishing of the bulk cosmological constant without any fine-tuning.

[2]While finishing this manuscript, we became aware of the paper [9] where such a phenomenon was argued to occur in a different brane world model (and in a different context) which contains two negative as well as two positive tension branes.

$(D-1)$-dimensional in the infra-red.

In this paper will illustrate the points raised above in a concrete model. The action for this model is given by:

$$S = \widehat{M}_P^{D-3} \int_\Sigma d^{D-1}x\sqrt{-\widehat{G}}\left[\widehat{R} - \widehat{\Lambda}\right] + M_P^{D-2}\int d^D x\sqrt{-G}\left\{R + \zeta\left[R^2 - 4R_{MN}^2 + R_{MNST}^2\right]\right\} .$$

(1)

For calculational convenience we will keep the number of space-time dimensions D unspecified. In (1) $\widehat{M}_P$ is (up to a normalization factor - see below) the $(D-1)$-dimensional (reduced) Planck scale, while M_P is the D-dimensional one. The $(D-1)$-dimensional hypersurface Σ, which we will refer to as the brane, is the $y = y_0$ slice of the D-dimensional space-time, where $y \equiv x^D$, and y_0 is a constant. Next,

$$\widehat{G}_{\mu\nu} \equiv \delta_\mu{}^M \delta_\nu{}^N G_{MN}\Big|_{y=y_0} ,$$

(2)

where the capital Latin indices $M, N, \ldots = 1, \ldots, D$, while the Greek indices $\mu, \nu, \ldots = 1, \ldots, (D-1)$. The quantity $\widehat{\Lambda}$ is the brane tension. More precisely, there might be various (massless and/or massive) fields (such as scalars, fermions, gauge vector bosons, *etc.*), which we will collectively denote via Φ^i, localized on the brane. Then $\widehat{\Lambda} = \widehat{\Lambda}(\Phi^i, \nabla_\mu \Phi^i, \ldots)$ generally depends on the vacuum expectation values of these fields as well as their derivatives. In the following we will assume that the expectation values of the Φ^i fields are dynamically determined, independent of the coordinates x^μ, and consistent with $(D-1)$-dimensional general covariance. The quantity $\widehat{\Lambda}$ is then a constant which we identify as the brane tension. Finally, the coefficient ζ has the dimension of length squared, and the term it multiplies is the Gauss-Bonnet term, which is quadratic in curvature. Also, note that we have set the D-dimensional bulk cosmological constant to zero, and there are no bulk fields other than gravity.

The model defined in (1) is a generalization of the Dvali-Gabadadze-Porrati model recently proposed in [7]. In fact, the difference between the two models (on top of the straightforward generalization that we do not *a priori* assume that the brane is tensionless) is the presence of the bulk Gauss-Bonnet term, which we have added in order to illustrate that higher derivative terms do not seem to modify our conclusions. The model defined in (1) reduces to the Dvali-Gabadadze-Porrati model for $\zeta = 0$.

Before we turn to our main point, let us briefly comment on the $\sqrt{-\widehat{G}}\widehat{R}$ term in the brane world-volume action. Typically such a term is not included in discussions of various brane world scenarios (albeit usually the $-\sqrt{-\widehat{G}}\widehat{\Lambda}$ term is included). However, as was pointed out in [7], even if such a term is absent at the tree level, as long as the brane world-volume theory is not conformal, it will typically be generated by quantum loops of other fields localized on the brane[3] (albeit not necessarily with the desired sign, which, nonetheless, appears to be as generic as the opposite one).

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[3]This is an important observation which might sometimes modify various conclusions, and should in principle be taken into account when discussing other brane world scenarios as well, for instance, in the Randall-Sundrum type of models [8].

An important feature of the above model is that (for the standard values of D) we can supersymmetrize the bulk action. Thus, in the following we will assume that the bulk is supersymmetric, and the bulk cosmological constant vanishes. On the other hand, as we have already mentioned, since the volume of the y dimension is infinite, supersymmetry on the brane could be completely broken, while bulk supersymmetry is intact. In the remainder of this paper we will address the question whether bulk supersymmetry protects the brane cosmological constant.

II. BULK SUPERSYMMETRY AND BRANE COSMOLOGICAL CONSTANT

To proceed further, we will need equations of motion following from the action (1). Here we are interested in studying possible solutions to these equations which are consistent with $(D-1)$-dimensional general covariance. That is, we will be looking for solutions with the warped metric of the following form:

$$ds_D^2 = \exp(2A)ds_{D-1}^2 + dy^2 , \tag{3}$$

where the warp factor A, which is a function of y, is independent of the coordinates x^μ, and the $(D-1)$-dimensional interval is given by

$$ds_{D-1}^2 = \widetilde{g}_{\mu\nu}dx^\mu dx^\nu , \tag{4}$$

with the $(D-1)$-dimensional metric $\widetilde{g}_{\mu\nu}$ independent of y. With this ansätz the equations of motion are given by:

$$\left\{ (D-1)(D-2)(A')^2 - \frac{D-1}{D-3}\widetilde{\Lambda}\exp(-2A) \right\} - (D-3)(D-4)\zeta \times$$
$$\times \left\{ (D-1)(D-2)(A')^4 - 2\frac{D-1}{D-3}\widetilde{\Lambda}(A')^2\exp(-2A) + \right.$$
$$\left. + \frac{\widetilde{\chi}}{(D-3)(D-4)}\exp(-4A) \right\} = 0 , \tag{5}$$

$$\left\{ (D-1)(D-2)A'' + \frac{D-1}{D-3}\widetilde{\Lambda}\exp(-2A) \right\} - 2(D-3)(D-4)\zeta \times$$
$$\times \left\{ (D-1)(D-2)A''(A')^2 - \frac{D-1}{D-3}\widetilde{\Lambda}\left[A'' - (A')^2\right]\exp(-2A) - \right.$$
$$\left. - \frac{\widetilde{\chi}}{(D-3)(D-4)}\exp(-4A) \right\} +$$
$$+ \frac{D-1}{2}L\left[\widehat{\Lambda} - \widetilde{\Lambda}\exp(-2A)\right]\delta(y-y_0) = 0 , \tag{6}$$

where

$$L \equiv \widehat{M}_P^{D-3}/M_P^{D-2} . \tag{7}$$

Here $\widetilde{\Lambda}$ is independent of x^μ and y. In fact, it is nothing but the cosmological constant of the $(D-1)$-dimensional manifold, which is therefore an Einstein manifold, corresponding

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to the hypersurface Σ. Our normalization of $\widetilde{\Lambda}$ is such that the $(D-1)$-dimensional metric $\widetilde{g}_{\mu\nu}$ satisfies Einstein's equations:

$$\widetilde{R}_{\mu\nu} - \frac{1}{2}\widetilde{g}_{\mu\nu}\widetilde{R} = -\frac{1}{2}\widetilde{g}_{\mu\nu}\widetilde{\Lambda} \ , \tag{8}$$

so that the $(D-1)$-dimensional Ricci scalar is given by

$$\widetilde{R} = \frac{D-1}{D-3}\widetilde{\Lambda} \ . \tag{9}$$

Moreover, the quantity

$$\widetilde{\chi} \equiv \widetilde{R}^2 - 4\widetilde{R}_{\mu\nu}^2 + \widetilde{R}_{\mu\nu\sigma\tau}^2 \tag{10}$$

is also a constant (if $\zeta \neq 0$).

Here we note that in the bulk (that is, for $y \neq y_0$) the second order equation (6) is automatically satisfied once the first order equation (5) is satisfied. As usual, this is a consequence of Bianchi identities.

For our purposes here it will not be necessary to find the most general solutions to the above equations. It will instead suffice to understand what are the restrictions on the warp factor coming from the requirement that the bulk be supersymmetric.

A. Killing Spinors and Bulk Supersymmetry

For the bulk to be supersymmetric, we must have covariantly constant Killing spinors satisfying the following equation (which comes from the requirement that the bulk gravitino ψ_M have a vanishing variation under the corresponding supersymmetry transformation):

$$\mathcal{D}_M \varepsilon = 0 \ . \tag{11}$$

Here ε is the Killing spinor, and $\mathcal{D}_M$ is the covariant derivative

$$\mathcal{D}_M \equiv \partial_M + \frac{1}{4}\Gamma_{AB}\omega^{AB}{}_M \ . \tag{12}$$

The spin connection $\omega^{AB}{}_M$ is defined via the vielbeins $e^A{}_M$ in the usual way (here the capital Latin indices $A, B, \ldots = 1, \ldots, D$ are lowered and raised with the D-dimensional Minkowski metric η_{AB} and its inverse, while the capital Latin indices $M, N, \ldots = 1, \ldots, D$ are lowered and raised with the D-dimensional metric G_{MN} and its inverse). Furthermore,

$$\Gamma_{AB} \equiv \frac{1}{2}\left[\Gamma_A \ , \ \Gamma_B\right] \ , \tag{13}$$

where Γ_A are the constant Dirac gamma matrices satisfying

$$\{\Gamma_A \ , \ \Gamma_B\} = 2\eta_{AB} \ . \tag{14}$$

Next, we would like to study the above Killing spinor equations in the warped backgrounds of the form (3):

$$\varepsilon' = 0 \ , \tag{15}$$

$$\widetilde{\mathcal{D}}_\mu \varepsilon + \frac{1}{2} A' \exp(A) \widetilde{\Gamma}_\mu \Gamma_D \varepsilon = 0 \ . \tag{16}$$

Here $\widetilde{\mathcal{D}}_\mu$ is the $(D-1)$-dimensional covariant derivative corresponding to the metric $\widetilde{g}_{\mu\nu}$, $\widetilde{\Gamma}_\mu$ are the $(D-1)$-dimensional Dirac gamma matrices satisfying

$$\left\{ \widetilde{\Gamma}_\mu \ , \ \widetilde{\Gamma}_\nu \right\} = 2\widetilde{g}_{\mu\nu} \ . \tag{17}$$

Also, note that Γ_D, which is the D-dimensional Dirac gamma matrix Γ_M with $M = D$ (that is, the Dirac gamma matrix corresponding to the $x^D = y$ direction) is constant in this background.

To begin with, note that (15) and (16) do not have a solution unless

$$A' \exp(A) = C \ , \tag{18}$$

where C is some constant. Let us assume that this condition is satisfied. We can rewrite the system of equations (15) and (16) as follows. Let

$$\gamma_\mu \equiv \widetilde{\Gamma}_\mu \Gamma_D \ . \tag{19}$$

These new gamma matrices satisfy

$$\{\gamma_\mu \ , \ \gamma_\nu\} = -2\widetilde{g}_{\mu\nu} \equiv 2\rho_{\mu\nu} \ , \tag{20}$$

that is, γ_μ are the gamma matrices for a space with the metric

$$\rho_{\mu\nu} = -\widetilde{g}_{\mu\nu} \ , \tag{21}$$

whose signature is $(+,-,\ldots,-)$. The Killing spinor equations now read (note that the covariant derivative $\widetilde{\mathcal{D}}_\mu$ is unaffected by the metric inversion)

$$\varepsilon' = 0 \ , \tag{22}$$

$$\widetilde{\mathcal{D}}_\mu \varepsilon + \frac{1}{2} C \gamma_\mu \varepsilon = 0 \ , \tag{23}$$

which have non-trivial solutions for the $\mathrm{AdS}_{D-1} \times \mathbf{R}$ space with the signature $(+,-,\ldots,-)$ and negative cosmological constant for the AdS_{D-1} piece given by

$$\lambda = -(D-2)(D-3)C^2 \ . \tag{24}$$

Note that under the metric inversion the Ricci scalar and, therefore, the cosmological constant flip their sign. This implies that the Killing spinor equations have a non-trivial solution provided that the metric $\widetilde{g}_{\mu\nu}$ on the brane corresponds to a de Sitter space with the signature $(-,+,\ldots,+)$ and positive cosmological constant

$$\widetilde{\Lambda} = -\lambda = (D-2)(D-3)C^2 = (D-2)(D-3)(A')^2 \exp(2A) \ . \tag{25}$$

Here we note that for such warp factors the bulk curvature, which is given by

$$R = \widetilde{R}\exp(-2A) - (D-1)\left[2A'' + D(A')^2\right] , \tag{26}$$

vanishes.

Next, the fact that the metric $\rho_{\mu\nu}$ is that of the AdS$_{D-1}$ space (with the signature $(+,-,\ldots,-)$) implies that the corresponding Riemann tensor

$$\mathcal{R}_{\mu\nu\sigma\tau} = \frac{\lambda}{(D-2)(D-3)}\left[\rho_{\mu\sigma}\rho_{\nu\tau} - \rho_{\mu\tau}\rho_{\nu\sigma}\right] . \tag{27}$$

Since the Riemann tensor flips sign under the metric inversion, we obtain

$$\widetilde{R}_{\mu\nu\sigma\tau} = \frac{\widetilde{\Lambda}}{(D-2)(D-3)}\left[\widetilde{g}_{\mu\sigma}\widetilde{g}_{\nu\tau} - \widetilde{g}_{\mu\tau}\widetilde{g}_{\nu\sigma}\right] . \tag{28}$$

This implies that $\widetilde{\chi}$ defined above is given by

$$\widetilde{\chi} = \frac{(D-1)(D-4)}{(D-2)(D-3)}\widetilde{\Lambda}^2 . \tag{29}$$

Now we can readily see that (5), as well as (6) in the bulk, are satisfied as long as we have bulk supersymmetry, which implies (25) and (29). As to the brane cosmological constant $\widetilde{\Lambda}$, it is related to the brane tension $\widehat{\Lambda}$ via the jump condition which follows from (6). The important point here is that it does not have to be zero to preserve bulk supersymmetry.

B. The Dvali-Gabadadze-Porrati Model

The last result seems to indicate that the aforementioned "zeroth-order" argument must somehow break down in the above model. Here we would like to better understand the precise mechanism for this breakdown. For simplicity we will do this in the Dvali-Gabadadze-Porrati model, that is, for $\zeta = 0$. Then for positive $\widetilde{\Lambda}$ we have the following bulk equation

$$(A')^2 \exp(2A) = C^2 . \tag{30}$$

Taking into account the jump condition which follows from (6), we obtain the following non-singular solution with infinite volume:

$$A(y) = \ln\left[C(y_+ - y)\right] , \quad y < y_0 , \tag{31}$$
$$A(y) = \ln\left[C(y - y_-)\right] , \quad y \geq y_0 , \tag{32}$$

where

$$y_\pm \equiv y_0 \pm \Delta , \tag{33}$$

and without loss of generality we have assumed $C > 0$. Note that the positive quantity Δ is fixed from the jump condition

$$\left\{\widehat{\Lambda} - \widetilde{\Lambda}\exp\left[-2A(y_0)\right]\right\} + 2(D-2)\frac{1}{L}\left[A'(y_0+) - A'(y_0-)\right] = 0 , \tag{34}$$

which can be rewritten as

$$\widehat{\Lambda} = (D-2)(D-3)\frac{1}{\Delta^2} - 4(D-2)\frac{1}{L\Delta} \ . \tag{35}$$

Let us discuss the possible solutions of this equation for Δ.

Thus, if $\widehat{\Lambda} > 0$, then the solution is given by (here we are assuming $L > 0$):

$$\Delta = \frac{2(D-2)}{\widehat{\Lambda}L}\left[\sqrt{1 + \frac{D-3}{D-2}\widehat{\Lambda}L^2} - 1\right] \ . \tag{36}$$

On the other hand, if $\widehat{\Lambda} < 0$, then we must have

$$|\widehat{\Lambda}| \leq \frac{D-2}{D-3}\frac{1}{L^2} \ . \tag{37}$$

We then have two solutions

$$\Delta_\pm = \frac{2(D-2)}{|\widehat{\Lambda}|L}\left[1 \pm \sqrt{1 - \frac{D-3}{D-2}|\widehat{\Lambda}|L^2}\right] \ . \tag{38}$$

Thus, we have a lower bound on the brane tension.

Here the following remark is in order. In the above solution the effective brane tension, defined as

$$f \equiv \widehat{\Lambda} - \widetilde{\Lambda}\exp[-2A(y_0)] \ , \tag{39}$$

is negative. Such a brane would suffer from world-volume ghosts unless we assume that it is an "end-of-the-world" brane located at an orbifold fixed point. Thus, in the above solution the geometry of the y dimension is that of $\mathbf{R}/\mathbf{Z}_2$ (and not of $\mathbf{R}$), with the orbifold fixed point identified with y_0 (note that the above solution has the $\mathbf{Z}_2$ symmetry required for the orbifold interpretation), and the brane is stuck at the orbifold fixed point. Note that there is another solution with positive effective brane tension[4], which is given by

$$A(y) = \ln\left[C(y - y_-)\right] \ , \quad y_- < y < y_0 \ , \tag{40}$$
$$A(y) = \ln\left[C(y_+ - y)\right] \ , \quad y_+ > y \geq y_0 \ . \tag{41}$$

In this solution[5] the volume of the extra dimension is finite as the latter is cut off by horizons located at $y = y_\pm$. Note that in such a solution the argument of [1,2] does not apply to

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[4]Since $f > 0$ in this case, *a priori* there is no need to restrict to the orbifold interpretation even though the solution does possess the corresponding $\mathbf{Z}_2$ symmetry.

[5]A $D = 4$ version of this solution can be found in [10].

begin with, and this is precisely why we focus on the above solution with negative effective brane tension[6].

Next, note that by rescaling the coordinates x^μ on the brane we can always set $\exp[A(y_0)] = 1$. This is equivalent to setting $C = 1/\Delta$. In this case the $(D-1)$-dimensional Planck scale (which is related to the $(D-1)$-dimensional Newton's constant arising in the $(D-1)$-dimensional Newton's law) is simply $\widehat{M}_P$, while the D-dimensional Planck scale is M_P. The effective $(D-1)$-dimensional cosmological constant (in the Particle Physics conventions) is given by

$$\widetilde{\Lambda}_{\text{eff}} \equiv \widetilde{\Lambda}\widehat{M}_P^{D-3} . \tag{42}$$

Its ratio to $\widehat{M}_P^{D-1}$ is then $\widetilde{\Lambda}/\widehat{M}_P^2 \sim 1/\Delta^2 \widehat{M}_P^2$. Let us assume that $\widehat{\Lambda} > 0$ (which is generically expected to be the case once brane supersymmetry is broken). Then it is not difficult to see that we have

$$\Delta \lesssim 1/\sqrt{\widehat{\Lambda}} . \tag{43}$$

This then implies that

$$\widetilde{\Lambda}/\widehat{M}_P^2 \gtrsim \widehat{\Lambda}/\widehat{M}_P^2 . \tag{44}$$

That is, the brane cosmological constant is at least[7] as large as the brane tension, which in a four dimensional theory is generically expected to be $\sim (\text{TeV})^4$ (in the Particle Physics conventions) assuming that the brane supersymmetry breaking scale is $\sim \text{TeV}$.

Thus, we have arrived at the same unpleasant generic lower bound on the cosmological constant as in the usual four dimensional effective field theory. Naturally, one would like to better understand how come the aforementioned "zeroth-order" argument does not hold in the Dvali-Gabadadze-Porrati model. To do this, we will need to study the bulk graviton spectrum in this model.

Normalizable Modes

From (1) it is not difficult to see that the norm of a bulk graviton mode is proportional to

$$||\widetilde{h}_{\mu\nu}||^2 \sim \int dy \exp[(D-3)A]\sigma^2 , \tag{45}$$

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[6]Actually, there is the third solution with vanishing effective brane tension $f = 0$, which is given (up to equivalence under the reflection around y_0) by $A(y) = \ln[C(y-y_-)]$, $y > y_-$. In this solution the space in the y direction is cut off by a horizon at $y = y_-$. As far as our discussion in this paper is concerned, this solution has essentially the same properties as the one with negative effective brane tension, which we will focus on in the following.

[7]Phenomenologically the crossover scale L is supposed to be quite large [7], so that we actually expect $\widehat{\Lambda} \gg 1/L^2$, and $\Delta \sim 1/\sqrt{\widehat{\Lambda}}$. Because of this, the aforementioned lower bound on the brane tension in the case of negative $\widehat{\Lambda}$ would seem to require quite a bit of fine-tuning.

where $\sigma = \sigma(y)$ depends only on y. Moreover, $\sigma(y)$ satisfies the following equation [11,12]:

$$(\exp[(D-1)A]\sigma')' + m^2 \exp[(D-3)A]\sigma = 0 ,\qquad(46)$$

where m^2 is the mass squared of the corresponding graviton mode.

To study normalizability of these graviton modes, let us make the coordinate transformation $y \to z$ so that the metric takes the form:

$$ds_D^2 = \exp(2A)\left(\tilde{g}_{\mu\nu}dx^\mu dx^\nu + dz^2\right) .\qquad(47)$$

That is,

$$dy = \exp(A)dz ,\qquad(48)$$

where we have chosen the overall sign so that $z \to \pm\infty$ as $y \to \pm\infty$. We will conveniently choose the origin for the z coordinate to correspond to $y = y_0$. Then we have

$$z = -\frac{1}{C}\ln\left[\frac{y_+ - y}{\Delta}\right] , \quad y < y_0 \qquad(49)$$

$$z = +\frac{1}{C}\ln\left[\frac{y - y_-}{\Delta}\right] , \quad y \geq y_0 .\qquad(50)$$

We then have

$$A(z) = C|z| + \ln(C\Delta) .\qquad(51)$$

In terms of the z coordinate we have

$$\sigma_{zz} + (D-2)A_z\sigma_z + m^2\sigma = 0 ,\qquad(52)$$

where the subscript z denotes derivative w.r.t. z. Let

$$\sigma \equiv \exp\left[-\frac{1}{2}(D-2)A\right]\hat{\sigma} .\qquad(53)$$

Then the norm of $\tilde{h}_{\mu\nu}$ is given by

$$||\tilde{h}_{\mu\nu}||^2 \sim \int dz\hat{\sigma}^2 .\qquad(54)$$

This implies that for a given mode to be plane-wave/quadratically normalizable, $\hat{\sigma}$ must be plane-wave/quadratically normalizable w.r.t. the flat z coordinate.

The equation for $\hat{\sigma}$ reads:

$$\hat{\sigma}_{zz} + \left[m^2 - \frac{1}{2}(D-2)A_{zz} - \frac{1}{4}(D-2)^2(A_z)^2\right]\hat{\sigma} = 0 .\qquad(55)$$

Using (51) we obtain

$$\hat{\sigma}_{zz} + \left[\mu^2 - 2m_*\delta(z)\right]\hat{\sigma} = 0 ,\qquad(56)$$

where

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$$\mu^2 \equiv m^2 - m_*^2 \ , \tag{57}$$

and

$$m_* \equiv \frac{1}{2}(D-2)C \ . \tag{58}$$

For $m^2 > m_*^2$ the solution of (56) reads

$$\widehat{\sigma}(z) = \text{const.} \times \left[\cos(\mu z) + \frac{m_*}{\mu} \sin(\mu|z|) \right] \ . \tag{59}$$

Thus, the bulk gravitons with $m^2 > m_*^2$ are plane-wave normalizable. For $m^2 = m_*^2$ the solution reads:

$$\widehat{\sigma}(z) = \text{const.} \times [1 + m_*|z|] \ , \tag{60}$$

so that this mode is not normalizable. Finally, for $m^2 < m_*^2$ the solution is given by

$$\widehat{\sigma}(z) = \text{const.} \times \left[\cosh\left(\sqrt{-\mu^2}\,z\right) + \frac{m_*}{\sqrt{-\mu^2}} \sinh\left(\sqrt{-\mu^2}|z|\right) \right] \ , \tag{61}$$

which is not plane-wave normalizable. Thus, we see that if the cosmological constant on the brane is positive, then we have a *mass gap* in the spectrum of the bulk modes. Once again, let us set $C = 1/\Delta$ (so that the $(D-1)$-dimensional Planck scale is given by $\widehat{M}_P$). Then we have plane-wave normalizable bulk gravitons with masses larger than

$$m_* = \frac{D-2}{2}\frac{1}{\Delta} \gtrsim \sqrt{\widehat{\Lambda}} \ . \tag{62}$$

That is, the theory is actually $(D-1)$ dimensional at distance scales $r \gtrsim r_*$, where

$$r_* \sim 1/m_* \ . \tag{63}$$

Now suppose that $L \ll r_*$. Then at distance scales $r \lesssim L$ the theory is effectively $(D-1)$ dimensional, at scales $L \lesssim r \lesssim r_*$ the theory is D-dimensional, and, finally, at scales $r \gtrsim r_*$ it is $(D-1)$-dimensional again. This explains how come bulk supersymmetry does not protect the brane cosmological constant - the $(D-1)$-dimensional effective field theory is a good approximation below the energy scales $\sim m_*$. On the other hand, if $r_* \lesssim L$, then the theory is never D-dimensional but is always $(D-1)$-dimensional. Note that without fine-tuning in the phenomenological context we actually expect $r_* \ll L$.

Thus, to obtain a small cosmological constant on the brane we must fine-tune the brane tension. Note that in such a brane world model the present day cosmological evolution on the brane at scales comparable with the size of our Universe would be described by the four dimensional laws of gravity. In fact, this is expected to be the case even at earlier evolutionary stages such as inflation. On the other hand, the five dimensional nature of the model would have to show up at somewhat lower scales of order of the crossover distance scale L.

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